# Machine Learning 202 

Recommender Systems

## Outline

- Background
- Collaborative Filtering for 0-1 Data
- User based CF
- Item based CF
- Association Rules
- Evaluation of "top- N " recommender algo
- Examples using recommenderlab from cran r on MS weblogs


# Netflix Problem 

- Customer logs onto Netflix site
- Has known history w Netflix
- Past movie ratings
- Movies watched
- What movies should Netflix promote to the user?


## Netflix Prize

- Netflix had a system in place to predict how a user would rate movies they hadn't seen.
They wanted better performance
- In 2006 Netflix decided to have a contest
- They offerred $\$ 1$ million to first person (or team) that could improve upon Netflix prediction performance by $10 \%$


## Scale of the problem and the contest

- 480,000 users 17,770 movies, 100 million ratings
- 6 years of data 2000 through 2005
- 2700 teams enter competition
- 3 years to finish.


## Recommendation - Ask your Brother

- Find people with similar tastes and ask them for recommendations
- Called "Collaborative Filtering"
- Transaction-based
- Characterize movies based on who gives them the same ratings.


## Collaborative Filtering

- Here's a table of ratings

| User1 | User2 | User3 |
| :---: | :---: | :---: |
| 4 | 3 | 0 |
| 5 | 3 | 1 |
| 0 | 0 | 5 |

- Movie1 is closer to Movie2 than it is to Movie3, based on user ratings. (and User1 is closer to User2)


## Collaborative Filtering - Binary Data

- Suppose that all we have is data on what movies were watched
$\begin{array}{llll}\text { Movie1 } & 1 & 1 & 0\end{array}$
Movie2 $1 \quad 1 \quad 1$
$\begin{array}{llll}\text { Movie3 } & 0 & 0 & 1\end{array}$
- Without the rating proximity is obvious
- Binary data are completely objective


## Collaborative Filtering

- This is significantly more precise than attribute-based
- Don't need to tell it which attributes are important
- Exploits judgments of other users
- Not limited by user's self designated profile
- Not limited by movie's self-reported profile

And

- Transaction records become a source of competitive advantage!


## Other Problems Amenable to this Approach

- Movies
- Based on Movies Watched (versus ratings)
- Books (electronics, cameras, etc)
- Based on Purchase Transactions (Amazon, ebay, etc.)
- Ad serving
- Based on Ads clicked
- double click (do you auto-delete the dc cookie?)
- google (are you signed in?)
- Others?


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## Collaborative Filtering on 0-1 Data

- Set of users - $\mathrm{U}=\{\mathrm{u} 1, \mathrm{u} 2, \ldots, \mathrm{um}\}$
- Set of items - I = \{i1, i2, ... , in $\}$
- Matrix of ratings, or 0-1's
$-\mathrm{R}=\{\mathrm{rij}\}$
- rij = $1 \quad$ if user i has preference for item j
$=0$ otherwise
- See any problems?


## 0 is different from 1

- An entry of " 1 " in the matrix means interest (or click or purchase, etc.)
- What does a "0" mean?
- User not aware of product
- User hasn't wanted it up to this point in time
- User dislikes product
- One-class data (recall using one class svm for fraud detection)


## What to do with " 0 "

- Usually don't have data to distinguish the different reasons for inaction (not clicking a link, etc.)
- Could use one-class tech
- Usually treat different meanings as a single class - results legitimize this approach


## Problem Formulation

- For user "a" ua $\in U$ (called the "active" user)
- Let la $=1 \backslash\{i l \in I$ such that ral $=1\}$ la is the set of items not selected by user "a"
- Predict ratings for all elements of la or
- Create a list of top N recommendations


## Types of Algorithms

- Memory-based - Search whole data base to develop ordered set of recommendations
- User-based CF
- Scalability problem
- Model-based - Use db to learn compact representation of answers


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## User-based CF

- Mimics word of mouth
- Find a neighborhood of users with similar tastes
- Neighborhood defined by similarity (or distance) measure
- Pearson correlation
- Cosine similarity
- Jacard similarity


## Similarity Measures

- Pearson correlation
$\mathrm{Sp}(\mathrm{x}, \mathrm{y})=\frac{\sum_{i \in I}(x i-x \operatorname{avg}) *(y i-y \operatorname{avg})}{(|I|-1) * s d(x) * s d(y)}$
- Cosine similarity
$\mathrm{Sc}(\mathrm{x}, \mathrm{y})=\frac{x \cdot y}{\|x\|^{2}| | y| | 2}$
- Jacard similarity

$$
\mathrm{Sj}(\mathrm{x}, \mathrm{y})=\frac{|X \cap Y|}{|X \cup Y|}
$$

## Develop Ratings for la

- Use similarity measure (or metric) to define a neighborhood N of ua (active user).
- Basically average the other user's ratings to estimate ua's rating.


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## Item-Based CF

- Model Building - Build an item-item similarity matrix - S
- Normalize $S$ so that rows sum to 1 .
- For each row (item) set to zero all but the largest similarities (to reduce model size)
- For each item calculate score by adding together similarity with active user's items
- Remove items already in actives user's set


## Item-based CF

- More efficient for computer time and storage than user-based
- Only slightly inferior in performance
- Successfully applied to large-scale problems (e.g. Amazon)


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## CF Using Association Rules

- What are association rules?
- Let $\mathrm{I}=\{\mathrm{i} 1, \mathrm{i} 2, \ldots, \mathrm{in}\}$ be a set of items (peanut butter, jelly, etc)
- Let $\mathrm{D}=\{\mathrm{t} 1, \mathrm{t} 2, \ldots \mathrm{tm}\}$ be a set of transactions
- Each ti a subset of I-(shopping cart)
- Association rule is an implication of the form: $X=>Y$ where $X, Y$ are both subsets of $I$ and $X \cap Y=\varnothing$ (chips => dip)


## Support and Confidence

- Support - For a set of items A subset of I support is support(A) $=\mid\{\mathrm{ti} \mid A$ is subset of ti$\}|/|D|$
- Support for an a-rule - For disjoint sets $X, Y$ (subsets of $I$ ) support( $\mathrm{X}=>\mathrm{Y}$ ) $=\operatorname{support}(\mathrm{XuY})$
- Confidence confidence $(\mathrm{X}=>\mathrm{Y})=$ support $(\mathrm{XuY}) /$ support $(\mathrm{X})$


## A-Rules for DF

- Treat each user's 1's as a single transaction
- Calculate rules of the form $X=>Y$ with highest confidence
- For X's that are subsets of active user's chosen items look up Y's and rank by confidence


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## Evaluation of Top-N recommender algorithms

- Given matrix R -
- Partition R - some rows for "test_set" the rest for "train_set"
- Train also on train_set
- Test performance on test_set
- For testing
- Treat each user as "active" user
- Remove some of user's actual selections
- See if given Top-N recommender algo replaces removed selections


## How to Split R

- Simple Split (for large data)
- Pick a reasonable fraction ( $30 \%$ test, $70 \%$ train)
- Sample at random
- Bootstrap Sampling - (for small data)
- Sample with replacement to form training set
- Test on users not included in training set
- k-fold Cross-Validation
- Divide users into k equal groups
- Run $k$ training/testing passes holding out a different one of $k$ groups for testing on each pass


## Delete Items for Test Users

- "Given j " - Keep " j " transactions and build recommender to fill in the others
- "All but j" - Delete "j" transactions


## Evaluating Performance

- For each user in test set generate Top-N recommendations
- Build confusion matrix:

| Actual/Predicted | Negative | Positive |
| :---: | :---: | :---: |
| Negative | a | b |
| Positive | c | d |

- Notice b+d = N, c+d = \# withheld
- Some Performance Terms

Accuracy $=(a+d) /(a+b+c+d)$
Precision $=d /(b+d)$
Recall $=d /(c+d)$
TPR = Recall
FPR $=b /(a+b)$

## Discussion re Evaluation

- To evaluate performance can use ROC curve AUC and tools we discussed in ML 101
- This scheme doesn't distinguish between getting good recommendation at $1^{\text {st }}$ or $5^{\text {th }}$ in sequence - that may make a difference


## Singular Value Decomposition

- Suppose M is an mxn matrix
- Singular Value Decomposition of M is a product of matrices
$\mathrm{M}=\mathrm{U} \Sigma \mathrm{V}$ ' ( ' means matrix transpose)
where
$\mathrm{U}=\mathrm{mxm}$ unitary matrix ( $\mathrm{UU} \mathrm{U}^{\prime}=\mathrm{U} \mathbf{U}=\mathrm{I}$ )
$\Sigma=m \times n$ diagonal matrix of singular values - the singular
values are all positive and arrange in decreasing
magnitude
$\mathrm{V}^{\prime}=\mathrm{nxn}$ unitary matrix


## Low-Rank Approximation using SVD

- SVD can be used to generate low-rank approximations as follows.
- Suppose $\mathrm{M}=\mathrm{U} \mathrm{\Sigma} \mathrm{~V}^{\prime}$, as above. If we want an approximation to $M$ that is of rank $k$ (less that the rank of $M$ ).
- Form $\Sigma_{k}=\Sigma$ (with singular values smaller than the largest $k$ set to 0)
- Then $\mathrm{M}_{\mathrm{k}}=\mathrm{U} \Sigma_{\mathrm{k}} \mathrm{V}^{\prime}$ is the closest rank k approximation to M in the sense of Frobenius norm.


## How Does SVD Help?

- Think of SVD as finding an abstract concept space where the importance of concepts are indicated by the singular values
- U maps users into the concept space. V' maps items (movies, web pages, ads) into concept space.
- In concept space we can compare a movie and a user directly to one another.


## Calculate Similarity Using SVD

- Recall $\mathrm{M}=\mathrm{U} \mathrm{\Sigma} \mathrm{~V}^{\prime}$
- $M$ is $m x n$ (by convention $m=$ \#users, $n=\# i t e m s$ )
- Take a unit vector in item-space, call it $e_{i}$ (vector of O's except $i^{\text {th }}$ element which is 1 )
- Mei maps the $i^{\text {th }}$ item from item space to user space (the vector of users who selected the $i^{\text {th }}$ item)
- $\Sigma \mathrm{V}^{\prime} \mathrm{e}_{\mathrm{i}}$ is a column vector in concept space that represents the $\mathrm{i}^{\text {th }}$ item.


## Calculate Similarity Using SVD

- Users are represented by a vector in item-space (vector with 1's where corresponding to items of interest)
- Items are represented by a vector in item-space ( $\mathrm{e}_{\mathrm{i}}$ )
- Map the user and the items to concept-space using truncated SVD ( $\Sigma_{\mathrm{k}} \mathrm{V}^{\prime}$ ) and compare using directional similarity like correlation

